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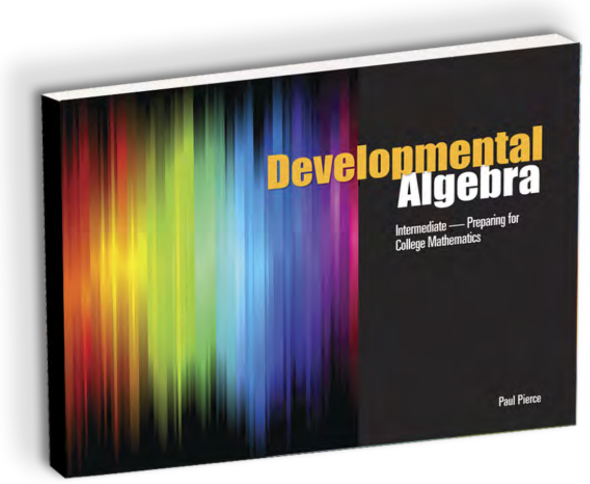

Developmental Algebra: Intermediate—Preparing for College Mathematics

By Paul Pierce

Included in this preview:

- Copyright Page
- Table of Contents
- Excerpt of Chapter 1

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Intermediate ALGEBRA

Preparing for College Mathematics

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Table of Contents

Dedication

To my beautiful wife, Laura. For over twenty years, she has held my hand while the lights have grown dim.

Chapter 1

Factoring Polynomials

A General Strategy for Factoring Polynomials

1. Always factor out the GCF first, if one exists. (Section 1.1)

2. How many terms does it have?

Four terms: Try factoring by grouping. (Section 1.2)

Three terms:

For $x^2 + bx + c$ try method from Section 1.3

For $ax^2 + bx + c$ try a*c*-method from Section 1.4

Two terms: Is it a difference of squares, difference of cubes or sum of cubes. (Section 1.5)

3. Always *factor completely*, and place parentheses around each **factor**.

4. Check by multiplying.

1.1 Greatest Common Factors (GCF)

a. Factor greatest common factor, GCF.

Factoring Terminology

To **factor** a polynomial means to write it as a product.

A **factor** of a polynomial is a polynomial that can be used to write the first polynomial as a product.

A **factorization** of a polynomial is a product that represents that polynomial.

A **prime factorization** of a polynomial is a factorization where each factor is prime.

GCF – Greatest Common Factor

How To Find the GCF of Two or More Monomials

1. Write the *prime factorization* of the coefficients, including –1 as a factor if any coefficient is negative.
2. Identify any common prime factors of the coefficients. For each one that occurs, include it as a factor of the GCF. If none occurs, use 1 as a factor.
3. Examine each of the variables as factors. If any appear as a factor of all the monomials, include it as a factor, using the smallest exponent of the variable. If none occurs in all the monomials, use 1 as a factor.
4. The GCF is the product of the results of steps (2) and (3).

Example 1

Find the GCF of 63 and 70.

Solution:

Write the prime factorization of each number.

$63 = 3 \cdot 3 \cdot 7$

$70 = 2 \cdot 5 \cdot 7$

The GCF is (7). ◄

Example A

Find the GCF of 30 and 125.

Example 2	Example B
Find the GCF of $42x^6$ and $105x^2$. Solution: Write the prime factorization of each number. $42x^6 = 2 \cdot 3 \cdot 7 \cdot x^2 \cdot x^4$ $105x^2 = 3 \cdot 5 \cdot 7 \cdot x^2$ The GCF is $(3)(7)(x^2) = (21x^2)$ ◄	Find the GCF of $42x^5$ and $28x^3$.
Example 3	**Example C**
Find the GCF of $27x^6y^5$, $36x^3y^4$, $-54x^4y^3$, and $99x^5y$. Solution: Prime factorizations. $27x^6y^5 = 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x\ \ y^5$ $36x^3y^4 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x\ \ y^4$ $-54x^4y^3 = -1 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x\ \ y^3$ $99x^5y = 3 \cdot 3 \cdot 11 \cdot x \cdot x \cdot x \cdot x \cdot x\ \ y$ The GCF of the coefficients is (3)(3) = (9). The GCF of the variables is (x^3y), since 3 is the smallest exponent of x, and 1 is the smallest exponent of y. GCF = $(9x^3y)$ ◄	Find the GCF of $30x^4y$, $-48x^3y^2$, $54x^5y^5$ and $12x^2y^3$.

"Factoring-out" the GCF of a Polynomial

If there is a GCF contained in each term of a polynomial, the GCF can be "factored out" using the distributive property: $ab + ac = a(b + c)$.

Multiply

$(3x)(x^2 + 2x - 5)$

$= (3x)(x^2) + (3x)(2x) - (3x)(5)$

$= 3x^3 + 6x^2 - 15x$

Factor

$3x^3 + 6x^2 - 15x$

$= (3x)(x^2) + (3x)(2x) - (3x)(5)$

$= (3x)(x^2 + 2x - 5)$

IMPORTANT: The number of terms in the parentheses MUST be the same as the number of terms in the original polynomial.

Think of factoring as the opposite of multiplying. This means factoring is the same as dividing! Factoring is NOT subtracting!!!

Example 4

Factor $21x - 35$.

Solution:

$21x - 35 = (7)(3)(x) - (7)(5)$

The GCF is (7). Since factoring a GCF is the same as dividing, we can rewrite $21x - 35$ as

$$21x - 35 = (7)\left(\frac{21x}{7} - \frac{35}{7}\right)$$

$$= (7)(3x - 5) \quad ◄$$

Check: $(7)(3x - 5) = (7)(3x) - (7)(5) = 21x - 35$

Example D

Factor $15x - 20$.

Example 5

Factor $10x^8 + 26x^3$.

Solution:

$10x^8 + 26x^3 = (2)(5)(x^3)(x^5) + (2)(13)(x^3)$

$= (2x^3)(5x^5) + (2x^3)(13)$

The GCF is $(2x^3)$.

$= (2x^3)(5x^5 + 13) \quad ◄$

Example E

Factor $24x^7 + 15x^4$.

Example 6

Factor $45x^6y^7 + 9x^5y^6$

Solution:

The GCF is $(9x^5y^6)$.

$$45x^6y^7 + 9x^5y^6 = (9x^5y^6)(5xy) + (9x^5y^6)(1)$$

$$= (9x^5y^6)(5xy + 1) \quad ◀$$

Example F

Factor $14x^4y^5 - 7x^3y^4$

Example 7

Factor $6x^5 + 9x^4 - 12x^3$

Solution:

$$6x^5 + 9x^4 - 12x^3 = (3x^3)(2x^2) + (3x^3)(3x) - (3x^3)(4)$$

The GCF is $(3x^3)$.

$$= (3x^3)(2x^2 + 3x - 4) \quad ◀$$

Example G

Factor $20x^4 + 35x^3 - 40x^2$

Example 8

Factor $-4xy + 20xz - 8x$

Solution:

When the leading coefficient of the polynomial is negative, factor out -1 as part of the GCF. The reason for this will be explained in the next section.

The GCF is $(-4x)$.

$$-4xy + 20xz - 8x = (-4x)(y - 5z + 2) \quad ◀$$

$$\boxed{\frac{-4xy}{-4x}} \quad \boxed{\frac{20xz}{-4x}} \quad \boxed{\frac{-8x}{-4x}}$$

Example H

Factor $-3xy - 9xz + 12x$

Exercise Set 1.1

For exercises 1-24, factor-out the GCF.

1) $3x - 15$

2) $7x - 21$

3) $5x^2 + 25x$

4) $5x^2 + 10x$

5) $9x^9 - 12x^4 - 27x^2$

6) $14x^8 + 8x^6 + 8x^4$

7) $64x^8 - 144x^4 - 128x^2$

8) $24x^9 - 60x^7 - 72x^3$

9) $17x^2 - 20y^3$

10) $25x^2 - 21y^3$

11) $3x^2y^7 + 27x^2y^6$

12) $3x^2y^4 + 12x^2y^3$

13) $30x^8y^8 - 48x^6y^5 - 30x^3y^3$

14) $40x^9y^9 + 36x^4y^6 + 8x^2y^3$

15) $x^8 - 3xy^3 + 3x^3y^7 - 47x^8y^3$

16) $x^4 - 11xy^4 + 11x^3y^5 - 41x^4y^4$

17) $\frac{5}{3}x^{10} - \frac{8}{3}x^9 + \frac{4}{3}x^8 + \frac{1}{3}x^7$

18) $\frac{5}{3}x^7 - \frac{5}{3}x^6 + \frac{8}{3}x^5 + \frac{1}{3}x^4$

19) $5x(4x - 5) - 3(4x - 5)$

20) $4x(2x - 5) + 5(2x - 5)$

21) $x(9 - z) + y(9 - z)$

22) $x(5 - z) + y(5 - z)$

23) $7x(9 - x) + 9y(9 - x)$

24) $9x(4 - x) + 4y(4 - x)$

1.2 Factoring by Grouping

a. Factor polynomials by grouping.

Factoring by Grouping

To factor a polynomial with four terms:

1. Factor out the GCF of all four terms.
2. Group the first two terms together and factor the GCF of those two terms.
3. Group the last two terms together and factor the GCF of those two terms.
4. Factor out the GCF of the two groups.

Although this procedure is described here for a polynomial with four terms, it can be "tried" with any polynomial with four or more terms.

Example 1

Factor $3x^3 + 9x^2 + x + 3$ by grouping.

Solution:

$$3x^3 + 9x^2 + x + 3 = \overbrace{3x^3 + 9x^2}^{GCF\text{ is }(3x^2)} + \overbrace{x + 3}^{GCF\text{ is }(1)}$$

$$= (3x^2)(x + 3) + (1)(x + 3)$$

$$= (x + 3)(3x^2 + 1) \blacktriangleleft$$

Example A

Factor $2x^3 + 8x^2 + 13x + 52$ by grouping.

Example 2

Factor $10x^3 + 6x - 20x^2 - 12$ by grouping.

Solution: Factor out the GCF of (2), then factor by grouping.

$$10x^3 + 6x - 20x^2 - 12 = (2)(5x^3 + 3x - 10x^2 - 6)$$

$$= (2)(5x^3 - 10x^2 + 3x - 6)$$

$$= (2)[\underline{5x^3 - 10x^2} + \underline{3x - 6}]$$

$$= (2)[(5x^2)(x - 2) + (3)(x - 2)]$$

$$= (2)(x - 2)(5x^2 + 3) \blacktriangleleft$$

Example B

Factor $9x^4 - 27x^3 + 3x - 9$ by grouping.

Example 3

Factor $6x^2 - 15x - 4x + 10$ by grouping.

Solution:

$$6x^2 - 15x - 4x + 10 = \overbrace{6x^2 - 15x}^{GCF \text{ is } (3x)} \; \overbrace{-4x + 10}^{GCF \text{ is } (-2)}$$

Notice that the GCF in the second group is negative, because the subtraction sign is included with the second grouping.

Therefore, remember to factor-out the negative, which changes the signs of the last two terms.

$$= (3x)(2x - 5) + (-2)(2x - 5)$$

$$= (2x - 5)(3x - 2)$$ ◄

Example C

Factor $4x^2 - 18x - 6x + 27$ by grouping.

Exercise Set 1.2

For exercises 1-14, factor by grouping.

1) $x^3 + 7x^2 + 6x + 42$

2) $x^3 + 5x^2 + 4x + 20$

3) $x^3 + 7x^2 - 3x - 21$

4) $x^3 + 6x^2 - 6x - 36$

5) $x^3 - 8x^2 - 5x + 40$

6) $x^3 - 4x^2 - 3x + 12$

7) $x^3 - 2x^2 + 6x - 12$

8) $x^3 - 2x^2 + 4x - 8$

9) $2x^3 - 18x^2 - 6x + 54$

10) $2x^3 - 8x^2 - 5x + 20$

11) $6x^6 + 10x^3 - 9x^3 - 15$

12) $12x^4 - 9x^2 + 20x^2 - 15$

13) $15x^2 - 20xy + 12xy - 16y^2$

14) $6x^2 - 15xy - 8xy + 20y^2$

1.3 Factoring Trinomials of the Form $x^2 + bx + c$

a. Factor trinomials of the form $x^2 + bx + c$

It's Just a Simple Number Game

In this section, we will be factoring trinomials of the form $x^2 + bx + c$ into products of the form $(x + \_)(x + \_)$, where we will need to find the numbers that fill-in the blanks. These numbers must satisfy certain criteria. Recall the FOIL method for multiplying two binomials. For example, $(x + 2)(x + 3) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$. The important thing to note is how the numbers 5 and 6 were obtained. The 6 is the ***product*** of 2 and 3, and the 5 is the ***sum*** of 2 and 3. So, looking at this example in reverse, the 2 and 3 which "fill-in the blanks" are the only two numbers which satisfy the two criteria: 1) their product is 6, and 2) their sum is 5. Thus, $x^2 + 5x + 6 = (x + 2)(x + 3)$.

Example 1

Factor $x^2 + 15x + 56$ and identify each factor.

Solution: Think of FOIL in reverse.

$$(x + \quad)(x + \quad)$$

Find two numbers that have a product of 56 and a sum of 15.

Factors of 56	Sums of Factors
1, 56	57
2, 28	30
4, 14	18
7, 8	**15**
−7, −8	−15
−4, −14	−18
−2, −28	−30
−1, −56	−57

Since $7 \cdot 8 = 56$ and $7 + 8 = 15$, the factorization is

$$(x + 7)(x + 8) \quad ◄$$

The ONLY two prime factors, other than 1, are the binomials $(x + 7)$ and $(x + 8)$. The variable x alone is not a factor, nor is 7, 8, 15, or 56.

To check, FOIL it out:

$$(x + 7)(x + 8) = x^2 + 8x + 7x + 56 = x^2 + 15x + 56$$

Example A

Factor $x^2 + 11x + 30$ and identify each factor.

Now try these:

$x^2 + 13x + 30 =$

$x^2 + 17x + 30 =$

$x^2 + 31x + 30 =$

How to Factor $x^2 + bx + c$ When c is POSITIVE

When the constant term is positive, look for two numbers with the same sign. The sign is that of the middle term:

$$x^2 - 7x + 10 = (x - 2)(x - 5)$$

$$x^2 + 7x + 10 = (x + 2)(x + 5)$$

Example 2

Factor $x^2 - 14x + 45$ and identify the factors.

Solution:

Since the constant term is positive and the coefficient of the middle term is negative, look for two negative numbers whose product is 45, and whose sum is –14.

Factors of 45	Sums of Factors
–1, –45	–46
–3, –15	–18
–5, –9	**–14**

$$x^2 - 14x + 45 = (x - 5)(x - 9) \quad ◀$$

The ONLY two prime factors are $(x - 5)$ and $(x - 9)$.

To check, simply multiply the two binomials:

$$(x - 5)(x - 9) = x^2 - 9x - 5x + 45 = x^2 - 14x + 45$$

Example B

Factor $x^2 - 10x + 24$ and identify the factors.

Now try these:

$x^2 - 25x + 24 =$

$x^2 - 14x + 24 =$

$x^2 - 11x + 24 =$

How to Factor $x^2 + bx + c$ When c is NEGATIVE

When the constant term of a trinomial is negative, look for two numbers whose product is negative. One must be positive and the other negative:

$$x^2 - 2x - 24 = (x + 4)(x - 6)$$

$$x^2 + 2x - 24 = (x - 4)(x + 6).$$

Select the two numbers so that the number with the larger absolute value has the same sign as b, the coefficient of the middle term.

Example 3

Factor $x^2 - 3x - 40$ and identify each factor.

Solution:

Since the constant term is negative, one number is negative and one is positive. Since the middle term is negative, the number with the larger absolute value is negative.

Factors of -40	Sums of Factors
1, –40	–39
2, –20	–18
4, –10	–6
5, –8	–3

$$x^2 - 3x - 40 = (x + 5)(x - 8) \quad ◄$$

The ONLY two prime factors are $(x + 5)$ and $(x - 8)$.

To check this answer, multiply:

$$(x + 5)(x - 8) = x^2 - 8x + 5x - 40 = x^2 - 3x - 40$$

Example C

Factor $x^2 - 4x - 21$ and identify each factor.

Example 4

Factor $x^2 - 35 + 2x$ and identify each factor.

Solution: Rewrite the trinomial in descending order: $x^2 + 2x - 35$.

Factors of -35	Sums of Factors
–1, 35	34
–5, 7	**2**

$$x^2 - 35 + 2x = x^2 + 2x - 35 = (x + 7)(x - 5) \quad \blacktriangleleft$$

The ONLY prime factors are $(x + 7)$ and $(x - 5)$.

To check, multiply:

$$(x + 7)(x - 5) = x^2 - 5x + 7x - 35 = x^2 + 2x - 35$$

Example D

Factor $x^2 - 27 + 6x$ and identify each factor.

Example 5

Factor $x^2 + 2xy - 35y^2$.

Solution:

The factors of $35y^2$, whose sum is $2y$, are $7y$ and $-5y$.

$$x^2 + 2xy - 35y^2$$

$$(x + 7y)(x - 5y) \quad \blacktriangleleft$$

The only difference between this example and the last one is the extra variable. The polynomial begins with an x^2, therefore each parenthesis begins with an x. For the same reason, since the polynomial ends with a y^2, each parenthesis ends with an y.

Example E

Factor $x^2 + 6xy - 27y^2$.

Example 6	Example F
Factor $4x^3 - 20x^2 + 24x$ completely, and identify each factor. Solution: Factor out the GCF of $(4x)$, then factor the resulting trinomial. $4x^3 - 20x^2 + 24x$ $(4x)(x^2 - 5x + 6)$ $(4x)(x \quad\;)(x \quad\;)$ We need two numbers whose product is 6 and whose sum is –5. These numbers are –3 and –2. $(4x)(x - 3)(x - 2)$ ◄ The three factors are $(4x)$, $(x - 3)$, and $(x - 2)$. Although $4x$ can be factored as $(2)(2)(x)$, and thus $(4x)(x - 3)(x - 2)$ can be written as $(2)(2)(x)(x - 3)(x - 2)$, it is usually preferable to leave it as $(4x)$.	Factor $2x^3 - 14x^2 + 24x$ completely, and identify each factor.

"Prime" Polynomials

A polynomial that cannot be factored is said to be **prime**.

Examples: $x^2 - x + 11$, $x^2 + 5$, $x - 3$

When factoring, always *factor completely*. This means the final factorization should contain only prime polynomials. The only exception to this would be if there was a monomial GCF which was not prime..

Exercise Set 1.3

For exercises 1-58, factor completely.

1) $x^2 - x - 30$

2) $x^2 - x - 56$

3) $x^2 + 9x - 22$

4) $x^2 + 5x - 84$

5) $x^2 - 7x - 30$

6) $x^2 - 5x - 66$

7) $x^2 - x - 45$

8) $x^2 - x - 63$

9) $7x^2 - 7x - 42$

10) $3x^2 - 3x - 18$

11) $2x^2 - 8x + 8$

12) $5x^2 - 35x + 50$

13) $40 - 3x - x^2$

14) $6 - 5x - x^2$

15) $x^2 + \frac{2}{5}x + \frac{1}{25}$

16) $x^2 + \frac{2}{7}x + \frac{1}{49}$

17) $x^2 - \frac{2}{9}x + \frac{1}{81}$

18) $x^2 - \frac{2}{5}x + \frac{1}{25}$

19) $x^2 + \frac{4}{3}x + \frac{4}{9}$

20) $x^2 + \frac{6}{7}x + \frac{9}{49}$

21) $x^2 + \frac{4}{7}x + \frac{4}{49}$

22) $x^2 + \frac{4}{3}x + \frac{4}{9}$

23) $x^2 + 1.6x + 0.64$

24) $x^2 + 1.2x + 0.36$

25) $x^2 + 1.4x + 0.49$

26) $x^2 + 0.8x + 0.16$

27) $x^2 + 0.5x + 0.06$

28) $x^2 + 1.3x + 0.42$

29) $x^2 + 0.2x - 0.08$

30) $x^2 + 0.2x - 0.48$

31) $x^3 - x^2 - 56x$

32) $x^3 - x^2 - 6x$

33) $x^3 + 7x^2 - 18x$

34) $x^3 + 2x^2 - 35x$

35) $x^8 - 3x^4 - 54$

36) $x^8 - 3x^4 - 70$

37) $2x^3 + 2x^2 - 40x$

38) $2x^3 + 4x^2 - 30x$

39) $x^4 - 15x^2 + 56$

40) $x^4 - 13x^2 + 40$

41) $5x^{11} - 10x^{10} + 120x^9$

42) $3x^7 - 3x^6 + 90x^5$

43) $5x^5 - 45x^4 - 70x^3$

44) $3x^{11} - 30x^{10} - 72x^9$

45) $x^2 + 5xy - 14y^2$

46) $x^2 + 3xy - 28y^2$

47) $x^2 - 4xy - 45y^2$

48) $x^2 - 2xy - 15y^2$

49) $x^2 - 5xy - 50y^2$

50) $x^2 - 4xy - 60y^2$

51) $x^2 + 7xy - 144y^2$

52) $x^2 + 3xy - 130y^2$

53) $x^2 + 2xy - 35y^2$

54) $x^2 + 4xy - 12y^2$

55) $x^2 - 6xy - 16y^2$

56) $x^2 - 3xy - 28y^2$

57) $x^2 - 2xy - 8y^2$

58) $x^2 - 5xy - 14y^2$

1.4 Factoring Trinomials of the Form $ax^2 + bx + c$, $a > 1$: The *ac*-Method

a. Factor trinomials of the Form $ax^2 + bx + c$, $a > 1$, Using The *ac*-Method

Although there are other methods of factoring this type of polynomial, the ac-method is an efficient procedure based on factoring by grouping (as shown in Section 1.2). This is not "trial and error" or "guess and check." It is a systematic approach with no guess work.

The ac-Method

As always, factor out any GCF, if any exists, before beginning any other procedure.

1. Multiply the leading coefficient a and the constant c, this is ac.
2. Find two integers such that their product is ac and their sum is b
3. Split the middle term. That is, write it as a sum or difference using the numbers found in step 2.
4. Factor by grouping. (Section 1.2)

Check by multiplying, as always.

Example 1

Factor $3x^2 + 19x + 28$ using the *ac*-method.

Solution:

There is no GCF.

1. Multiply the first number 3 and the last number 28:

$$ac = (3)(28) = 84.$$

2. Find two numbers whose product is 84 and whose sum is 19.

$$12 + 7 = 19 \qquad 12 \cdot 7 = 84$$

3. Rewrite the middle term as a sum:

$$3x^2 + 19x + 28 = 3x^2 + 12x + 7x + 28$$

4. Factor by grouping:

$$3x^2 + 12x + 7x + 28$$
$$(3x)(x + 4) + (7)(x + 4)$$
$$(x + 4)(3x + 7) \quad ◄$$

Check by multiplying.

$$(x + 4)(3x + 7) = 3x^2 + 7x + 12x + 28 = 3x^2 + 19x + 28$$

Example A

Factor $2x^2 + 11x + 15$ using the ac-method.

Now try these:

$2x^2 + 11x + 12 =$

$2x^2 + 11x + 14 =$

$3x^2 + 11x + 6 =$

Example 2

Factor $3x^2 + 16x - 35$ using the *ac*-method.

Solution:

There is no GCF.

1. Multiply the first number 3 and the last number -35:

$$ac = (3)(-35) = -105.$$

2. Find two numbers whose product is -105 and whose sum is 16.

$$-5 + 21 = 16$$

$$-5 \cdot 21 = -105$$

3. Rewrite the middle term as a sum:

$$3x^2 + 16x - 35$$

$$3x^2 - 5x + 21x - 35$$

4. Factor by grouping:

$$3x^2 - 5x + 21x - 35$$

$$(x)(3x - 5) + (7)(3x - 5)$$

$$(3x - 5)(x + 7) \quad \blacktriangleleft$$

Check by multiplying.

$$(3x - 5)(x + 7) = 3x^2 + 21x - 5x - 35 = 3x^2 + 16x - 35$$

Example B

Factor $6x^2 - x - 12$ using the ac-method.

Now try these:

$2x^2 - x - 15 =$

$7x^2 - x - 8 =$

$3x^2 - x - 14 =$